

# Joint maintenance and production planning of manufacturing systems subject to defective products\*

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## Abstract

In the past three decades, studies of joint maintenance and production planning have been focusing on age-dependent machine failure and inventory. This paper presents the interaction between defective products and optimal control of production rate, lead time and inventory. Our aim is to minimize the expected discounted overall cost due to maintenance activities, inventory holding and backlogs. We describe two operational and two maintenance states of a machine controlled by two decision variables: production and maintenance rates. The optimal policy is characterized by the dynamic programming solution to a piecewise deterministic optimal control problem. A numerical illustration is developed with a set of parameters calibrated on an existing manufacturing system.

*Key words:* Maintenance, Manufacturing systems, Stochastic dynamic programming, Numerical Methods

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## 1 Introduction

In today's global competitive marketplace, manufacturing industries seek to reduce costs due to machine failures. In fact, production planning is extensively linked with machine maintenance in many organizations. Several authors have developed optimal control models of manufacturing systems by integrating maintenance due to age-dependent machine failure (for example, see [1] and [3]). However, manufacturing systems are reliant on both machines and human being. Failure is not only due to the age of the machine but it is also due to human factor. Taking into account every failure parameter is quite difficult to achieve.

In this paper, we assume that failure causes can be linked to defective products. Thus, we propose an integrated maintenance and production planning focusing on defective product instead of age-dependent machine failure. Such interactions between defective and flawless products have been analysed in a manufacturing system [4, 5] but the integration of maintenance and production strategies has been mainly focused on machine and human activity [6, 7, 8]. Instead of taking into account machine failure and human error separately, we consider a defect in product as being the consequence of a combined failure; this consideration allows us to be more realistic by merging all failure parameters into a single one. Thus, a new stochastic control model which focuses on defective product, inventory and backlogs, has been developed. In this new model, we consider  $N$  operational states, one under repair and one preventive state. The transition between those states is controlled by a defect rate.

The remainder of this paper is organized as follow. First, we present a brief literature review in section 2 on integrated maintenance and production planning. In section 3 we define our integrated model of maintenance and production, then, we introduce the dynamic programming Hamilton-Jacobi-Bellman (HJB) equations. We then