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# Bivariate renewal function computational algorithm<sup>\*</sup>

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## Abstract

This paper deals with randomly failing single-unit systems with bivariate failure distributions considering for example time and usage. The development of replacement and warranty policies for such systems requires the knowledge and the computation of the bivariate renewal function which is impossible to obtain in closed form. A new numerical algorithm allowing the evaluation of this function is proposed for any bivariate renewal process. Numerical results are obtained and discussed using bivariate distributions based on stochastic functional relationships between the two variables, and other distributions displaying a correlation factor.

*Key words: Bivariate renewal process ,bivariate renewal function, bivariate replacement strategies*

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## 1 Introduction

In the last few decades, maintenance policies have been an active area of research. Valdez-Flores C. and Feldman R.M [1], Cho and Parlar [2], and Wang [3] have presented detailed surveys of much of the existing work on this subject. Maintenance problems can be classified according to the complexity of the systems (single-unit vs. multi-unit systems), their failure nature (self-announcing, non-self-announcing), the nature of associated maintenance actions (perfect vs. imperfect, instantaneous vs. non-instantaneous), and the performance criteria (cost rate over infinite or limited time span, system stationary or instantaneous availability).

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