A pseudo-polynomial time algorithm for the UET two-machine open-shop problem with two distinct time delays [*]

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Abstract

This paper discusses the problem of scheduling n unit execution time (UET) jobs with integral positive time delays on a two-machine open-shop environment. The criterion to minimize is the makespan. A pseudo-polynomial time algorithm is presented to solve the special case in which the time delays are restricted to two distinct values.

Key words: Makespan, Open-shop, Time delays, Well solvable case.

1 Introduction

This paper addresses the problem of scheduling a set $J = \{1, \ldots, n\}$ of n unit execution time (UET) jobs with time delay considerations on a two-machine open-shop environment. Each job comprises two operations: one operation has to be processed by one machine and the other by the second machine. The processing order of these two operations on the machines is not known in advance; it has to be determined when building the schedule. As usual, we assume the operations of a job cannot be processed at the same time, and a machine can only process at most one operation at a time. Furthermore, we assume a time delay, $\tau_i \in \mathbb{N}^+$, associated with each job $i \in J$, to denote the minimum time which must elapse between the completion of one of its operations and the start of the other operation. The goal is to seek a valid schedule which minimizes the makespan.

Motivation for the formulated problem comes from real applications. In traditional shop scheduling, it is a common practice to assume that once a job has finished one of its operations, it becomes immediately available for further processing. However, in many applications, this assumption is not justified. Indeed, there is often a significant time delay between the completion of an operation and the start of the next operation of the same job. In addition to delineating the border between polynomiality and intractability, in some cases, the execution times might be negligible compared to the time delays: assuming in this case

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